On the Role of Robustness in Multi-Objective Robust Optimization in the Design of Electromagnetic Devices

Armin Salimi[1], David A. Lowther[1]

[1]Department of Electrical and Computer Engineering, McGill University, Montreal, QC H3A 0E9, Canada,
armin.salimi@mail.mcgill.ca, david.lowther@mcgill.ca

This paper describes the different roles that robustness can assume when solving a multi-objective design optimization problem. A new role for robustness is proposed based on which an approach to conducting multi-objective robust optimization and finding the optima is suggested. This new approach is then tested on an analytical test problem and the results are presented.

***Index Terms*— Design Optimization, Pareto Optimization, Robustness**

I. Introduction

The performance of an electromagnetic design is usually affected by uncontrollable and/or unpredictable factors that cause perturbations to the performance values which were calculated during the design optimization process. Therefore, one of the highly desired characteristics of a good design is the insensitivity of its performance with respect to uncertainties. This concept of low sensitivity is the most popular approach to defining robustness. A design is called robust if it shows low sensitivity with respect to changes caused by these factors.

Robust optimization refers to the act of performing optimization when some degree of robustness is part of the criteria for optimality. Issues such as defining metrics for robustness and performing robust optimization have been well discussed under the context of single-objective optimization [1]-[2]; however, there has been little research work that deals with robustness in multi-objective problems and most of the existing work is extensions of measures and methods used in single-objective problems [3]. Considering that using most of the existing single-objective robustness measures and approaches in multi-objective problems would lead to significant increases in computational cost, it seems necessary to pay more attention to the concept of robustness from a multi-objective point of view.

The biggest problem of performing robust optimization is that robustness is usually very expensive to evaluate. This is an even bigger problem in electromagnetics since the cost of each solution evaluation is high. Therefore, finding ways of limiting the evaluation of robustness is very important. This paper is concerned with the different roles that robustness can play in multi-objective robust optimization.

II. Possible Roles of Robustness

The role that robustness assumes in the process of multi-objective robust optimization is usually one of the following:

A. *Robustness as extra objective(s)*

Suitable metrics can be chosen in order to measure robustness, which is then introduced into the objective space in the form of one or more objectives alongside the performance objectives.

B. *Robustness as a modifier to performance objectives*

There are multiple techniques for modifying performance objectives in order to account for robustness. The new robust performance objectives can then be used to perform regular optimization.

C. *Robustness as extra constraint(s)*

Measured values for robustness can be used as additional constraints which will reduce the size of the objective space.

D. *Robustness as modifier to dominance relations*

In some cases the definition of dominance is modified in order to account for robustness when comparing solutions in the objective space.

III. Proposed Role of Robustness

All of the possible roles of robustness mentioned in the previous section require the evaluation of robustness information on a wide range of the objective space. Considering that the evaluation of robustness is usually the most computationally expensive part in multi-objective robust optimization, it is very important that all types of unnecessary robustness calculations are avoided.

The assumption made here about the decision maker's preferences is that robustness is desirable only in optimal or close to optimal solutions and it is considered irrelevant in very sub-optimal solutions. It is also assumed that the trade-off information between robustness and performance is important and should be presented to the decision maker. This turns robustness into a semi-objective, meaning that it is treated as an objective only in the vicinity of the performance optima and ignored everywhere else. Putting robustness in this role has the advantage of keeping the useful trade-off information while avoiding increasing the dimensionality of the objective space and limiting the calculation of robustness to a sub-region of the objective space.

IV. Implementation

There is more than one way to use robustness in the proposed role. There needs to be a formulation which defines the region in the vicinity of the performance optima where the robustness information should be kept. This region is considered the optima in this context and the goal of the multi-objective robust optimization process is to find it. The implementation approach suggested here is based on creating a balance between the deterioration of performance and the gain in robustness as we move away from the Pareto front. In order to implement this balance two unary metrics are used. One is a sensitivity based metric called $PRHV_{dominated}$ [4] which can be used to measure the robustness of a solution and the other is the hyper-volume (*S*) [5] metric which is used to measure the degree of sub-optimality of a solution. Note that higher values of $PRHV_{dominated}$ correlate to lower robustness and vice versa. Two inequalities are used in order to define the optimal region:

$$PRHV_{Dominated} \leq PRHV_{Dominated}^{min} \tag{1}$$

$$\frac{S}{S_{ref}} \leq \alpha\left(1 - \frac{PRHV_{Dominated}}{PRHV_{Dominated}^{avr}}\right) \tag{2}$$

Figure 1 shows an arbitrary two dimensional objective space used to demonstrate the evaluation of (1) and (2) for the solution point specified with a cross. $PRHV_{Dominated}^{min}$ is the minimum value of $PRHV_{Dominated}$ among the solution points that dominate the specified solution point. S is the hyper-volume of the area enclosed by the Pareto front and the hyper-planes that contain the specified solution point. S_{ref} is the reference hyper-volume which is the hyper-volume of the area enclosed by the Pareto front and the hyper-planes that contain the nadir point. $PRHV_{Dominated}^{avr}$ is the average value of $PRHV_{Dominated}$ over the points that dominate the specified solution point. Also, α is a positive tunable parameter that controls the degree to which the trade-off between performance and robustness is allowed. Setting α to 0 reduces the optimal region to the Pareto front. It is recommended that α is set to 1 in order to avoid omitting any parts of the potentially valuable trade-off information of the optimal region. If and only if a solution points satisfies both (1) and (2) does it belong to the optimal region.

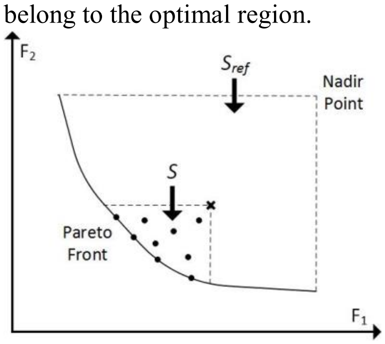

Fig. 1. Arbitrary two-dimensional objective space.

The optimization approach adopted here to perform multi-objective robust optimization consists of two steps. The first step is a generic multi-objective optimization run in order to find the Pareto front. The second step uses the end population of the first step as its initial population and tries to obtain a spread of points over the optimal region defined by (1) and (2).

V. Results and Conclusions

The suggested approach is applied to the analytical test problem defined and explained in [4] for $\alpha = 1$:

$$Minimize\ F = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{3}$$

$$\begin{cases} f_1(r,\theta) = \big(1 + g(r)\big)cos\theta \\ f_2(r,\theta) = \big(1 + g(r)\big)sin\theta \end{cases} \tag{4}$$

$$g(r) = r = 10x_2^3 - 15x_2^2 + 7.5x_2 \tag{5}$$

$$\theta = \frac{\pi}{2}x_1^5\,,\quad 0 \leq x_1, x_2 \leq 1 \tag{6}$$

Where f_1 and f_2 are objectives, x_1 and x_2 are design variables, and r and θ are intermediate parameters. As (5) and (6) suggest, heavy bias has been introduced into the mapping from the design space to the objective space of this test problem in order to build a simple test problem with variations in sensitivity across the objective space. The results of the first and the second step of the approach are shown in Fig. 2. Figure 3 uses a color map to show the values of robustness across the optimal region. These values are calculated through linear interpolation among the robustness values of the population solution points in the optimal region. As can be seen in Fig. 3, the optimal region shows a larger recession away from the Pareto front where the gain in robustness is higher. The results of applying this approach to an interior permanent magnet motor design problem will be presented in the extended paper.

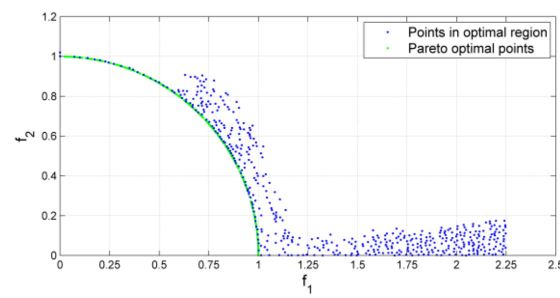

Fig. 2. Results of the two steps of multi-objective robust optimization.

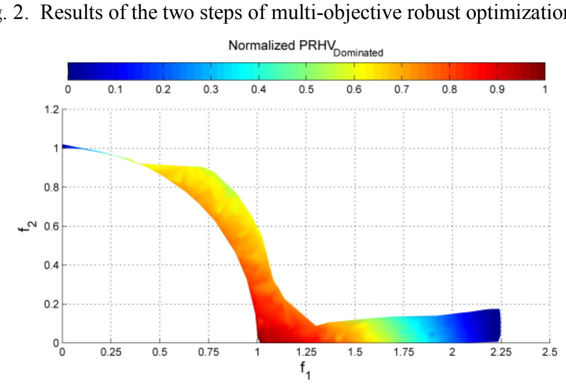

Fig. 3. Robustness across the optimal region.

VI. References

[1] R. T. Marler, J. S. Arora, "Robust optimization – a comprehensive survey" *Computer Methods in Applied Mechanics and Engineering*, vol. 196, no. 33-34, 2007, pp. 3190-3218.

[2] C. K. Goh, K. C. Tan, C. Y. Cheong, Y. S. Ong, "An investigation on noise-induced features in robust evolutionary multi-objective optimization" *Expert Systems with Applications*, vol. 37, no. 8, 2010, pp. 5960–5980.

[3] H. Beyer, B. Sendhoff, "Robust optimization – a comprehensive survey" *Computer Methods in Applied Mechanics and Engineering*, vol. 196, no. 33-34, 2007, pp. 3190-3218.

[4] A. Salimi, D. A. Lowther, "Multi-objective robust optimization of low frequency electromagnetic design problems using a new multi-objective robustness measure", *COMPEL - The international journal for computation and mathematics in electrical and electronic engineering*, submitted for publication.

[5] E. Zitzler, L. Thiele, "Multiobjective optimization using evolutionary algorithms-a comparative study", *Parallel Problem Solving from Nature*, Springer-Verlag, Amsterdam, pp. 292-301, September 1998.